An Analysis of Hedge Fund Performance Using Loess Fit Regression

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In this article, we analyze the returns distribution of Hedge Funds strategies, the average returns obtained over the past ten years and their correlation with a traditional portfolio. The aim is to identify the characteristics of each Hedge Fund investment strategy in order to be able to construct an optimal Hedge Fund portfolio for a Swiss pension fund. We will show that the classical linear correlation and the classical linear regression cannot be applied for Hedge Funds. Moreover, we will show that only three strategies, Convertible Arbitrage, Market Neutral and CTA, give diversification during market downturns. The techniques used are non-linear regressions and local correlations.

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Introduction

Previous research has questioned the use of simple linear regression models in describing the return relationship between hedge funds and the comparison asset market. In brief, while hedge fund may show evidence of diversification benefits over most market environments, various hedge fund strategies have shown to offer differing diversification benefits conditional on the performance of the stand alone stock and bond markets. In this paper the relationship between various hedge fund strategy returns and a Swiss based benchmark portfolio. Using a statistical methodology which captures non-linear relationships between the hedge fund strategy and the benchmark portfolio, we show that measuring the diversification benefit of investing in a Hedge Fund with the classical linear correlation coefficient is misleading.[1]

Hedge Fund Strategies

Hedge Fund indices differ widely in purpose, composition and weightings. The major differences relate to management staffing, performance determination and strategies. In the following section, we briefly analyze the most important investment strategies:

Convertible Arbitrage: involves purchasing a portfolio of convertible securities, generally convertible bonds and hedging a portion of the equity risk by short-selling the underlying common stock. Some managers may also seek to hedge interest rate exposures under certain circumstances. Most managers employ some degree of leverage, ranging from zero to 6:1.

Merger Arbitrage: funds invest simultaneously in long and short positions in both companies involved in a merger or acquisition. In stock swap mergers, the Hedge Funds are typically long the stock of the acquired company and short the acquiring company. In the case of a cash tender offer, the Hedge Funds are seeking to capture the difference between the tender price and the price at which the acquired company is traded. Profits are made by capturing the spread between the current market price of the target company and the price to which it will appreciate when the deal is completed. The risk is that the deal fails.

Emerging markets: funds invest in securities of companies or the sovereign debt of developing or “emerging” countries. This style is more volatile not only because emerging market are more volatile than developed markets, but because most emerging markets allow for only limited short selling and do not offer a viable futures contract to control risk. This suggests that Hedge Funds in emerging markets have a strong long bias.

Equity hedge: investing consists of a core holding of long equities hedged at all times with short sales of stocks and/or stock index options. The short position has three purposes. First, it is intended to generate alpha as well. Stock selection skill for short stocks can result in doubling the alpha. An equity hedge manager can add value by buying winners and selling losers. Second, the short position can serve the purpose of hedging market risk. Third, the manager earns interest on the short position.

Equity non-hedge: funds are predominately long term equities, although they have the ability to hedge with short sales of stocks and/or stock index options. The leverage is created by borrowing money or by using derivatives. Some strategies focus on long stock index futures or buying stocks, using them as collateral to borrow money (50%) which is then reinvested in more stocks.

Event driven: also known as "corporate life cycle" investing. This involves investing in opportunities created by significant transactional events, such as spin-offs, changes in ownership, bankruptcies, reorganizations, share-buy-backs and recapitalizations. The securities prices of the companies involved in these events are typically influenced more by the dynamics of the particular event than by the general appreciation or depreciation of the debt and equity markets.

Market Neutral: seek to profit by exploiting pricing inefficiencies between related equity securities, neutralizing exposure to market risk by combining long and short positions. Market neutral portfolios are designed to be either beta-neutral, currency-neutral or both.

Fixed Income: groups all strategies together, which can be performed with fixed income instruments like arbitrage, convertible-, diversified-, high yield- and mortgage bonds.

Macro: involves leveraged bets in liquid market on anticipated stock market price movements, interest rates, foreign exchange and physical commodities. They pursue a base strategy such as long/short or "future trend following" to which highly leverage bets in other markets are added a few times each year. They move from opportunity to opportunity and from trend to trend. Macro funds make their money by anticipating a price change early and not by exploiting market inefficiencies.

Short selling: involves the sale of a security not owned by the seller with the intention of buying it back later at a lower price. In addition the short seller earns interest on the cash proceeds from the short sale of stock. Given the extensive equity bull market, short selling strategies have not done well in the recent past. Technically, a short sale does not require an investment, but it does require collateral.

CTA: Commodity Trading Advisors are investing in commodity and financial futures. For example, two of the used techniques are long/short stock index futures based on quantitative or technical trend following indicator with stop loss limit, or stock index arbitrage. We include them in the analysis, even though they are not considered being Hedge Funds by the practitioners.

Data and Methodology

HFR data was used as the basis for the analysis which covers the period January 1990 till June 1999, based on monthly observations. The comparison index is constructed as the The LPP Index (BVG Index is the constructed by Pictet & Cie (Geneva) and represents the Benchmark Index for a Swiss institutional investor. Typically, this index does not include more than 30% of the SPI, 25% of the MSCI, 20% of the Salomon Brother Global Bond Index.)

The following performance measures were obtained and are shown in Exhibit 1.

μ : monthly mean returns (=ln (R(t)/R(t-1)))
σ : monthly standard deviation
S : skewness [2]
K : excess kurtosis [3]
R_{MAX} : maximum monthly returns over the period
R_{MIN} : minimum monthly returns over the period
$\rho_{LPP,\ BVG}$ [4]: linear correlation coefficient between the hedge fund strategy and the LPP Index

Exhibit 1

	μ	σ	*S*	*K*	R_{MAX}	R_{MIN}	ρ_{LPP}
Convertible Arbitrage	0.92%	1.04	-1.46	3.18	3.3%	-3.1%	0.44
Merger Arbitrage	1.00%	1.37	-3.21	13.7	2.9%	-6.4%	0.46
Emerging Markets	1.36%	4.64	-1.16	4.31	12.3%	-21.0%	0.59
Equity hedge	1.73%	2.36	-0.50	1.15	7.2%	-7.6%	0.51
Equity non-hedge	1.66%	3.83	-0.82	1.79	9.5%	-13.3%	0.59
Event Driven	1.35%	1.96	-1.78	7.34	5.1%	-8.9%	0.61
Market neutral	0.87%	0.90	-0.09	0.55	3.5%	-1.6%	0.07
Fixed income (Total)	0.98%	1.06	-0.58	6.05	5.3%	-3.2%	0.49
Macro	1.59%	2.67	0.10	0.16	7.8%	-6.4%	0.55
Short selling	0.22%	5.57	0.30	0.73	19.4%	-16.2%	-0.51
CTA	0.66%	2.76	0.44	0.35	10.0%	-5.5%	-0.16

First, all strategies achieve positive monthly mean returns. If we focus on a classical information ratio (i.e. mean returns divided by the standard deviation), the worst strategy is, by far, the short selling one, which is consistent with the stock market behavior of the last ten years. The best one is the market neutral, mainly because of its low level of standard deviation. If we look at the skewness and the kurtosis indicators, we observe that almost all strategies have a negative skewness[5] and a positive excess kurtosis, except for the macro, short-selling and the CTA strategies. This means that negative returns will deviate from normality, especially on the downside, except in the case of the macro and short-selling strategies. The Merger Arbitrage is deviating the most from normality, the skewness and the kurtosis being significant[6]. Finally, the linear correlation with the LPP/BVG portfolio is an important indicator for the investors. Many pension funds look for diversification benefits when they decide to invest in alternative instruments. Therefore, asset allocation advisors construct a portfolio with a low correlation level. With this objective in mind, the short selling, Convertible Arbitrage, CTA and market neutral strategies are interesting. The short selling strategy is an insurance, which some investors include in their portfolio. Like any other insurances, it has a price. In this case, the price consists of two factors, firstly the low level of return, at least when the markets of traditional instruments are bullish and secondly, the high level of standard deviation shown by the short selling strategy. It is interesting to observe that portfolio insurance can also be achieved by buying some put options, but not at the same costs and the same payoffs.

Loess Fit analysis

The objectives of this section are, firstly, to analyze the correlation between the LPP and the HFR indices using a methodology, which takes into account the non-linear relationship between both instruments and secondly, to analyze the payoff structure of the HFR indices. The methodology being used is the local regression analysis.

We shall show that four HFR strategies[7] out of nine result in concave payoffs as compared to the LPP index. This means that the slope of the local regression decreases, when the LPP index monthly returns are more positive. When LPP monthly returns are more negative, the HFR monthly returns are getting negative at an even higher proportional rate. Moreover, when the relation is statistically not linear between both series (as in the four case cited above), the classical correlation coefficient correlation is misleading and will, in the case of convexity relation, give a higher correlation during market downturns.

In the next section, the Loess fit analysis is conducted on ten hedge fund strategies and determine which ones are adding diversification to a Swiss pension fund portfolio.

Results

A local regression analysis on 9 different HFR investment styles (included an equally weighted Hedge Funds index) and the CTA strategy[8] are conducted. For each style selected, we also perform a Loess Fit analysis using a statistical software[9]. As noted above a Loess Fit is a technique, which displays local polynomial regressions with the bandwidth based on nearest neighbors. Briefly, for each data point in a sample, the software fits a locally weighted polynomial regression. It is a local regression since it uses only the subset of observations, which lies in the neighborhood of the point fitting the regression model. By using this technique, we are able to fit the non-linear relation between market returns and hedge funds returns. This technique increases the power of explanation of the regression and describes the non-linear relation between the market and each hedge fund.

We obtain a picture of local regressions between the LPP index and a hedge fund investment strategy. This picture helps us to identify the way to do the regression, that is, with the help of the standard linear regression, by means of a quadratic regression or finally with aid of a polynomial third degree regression. The significance of the local regressions[10] is verified using the adjusted R^2. The adjusted R^2 gives us the explanatory power of the local regression taking into account the number of independent variables. In our case, the independent variable is always the LPP Swiss Index. Therefore, the higher the adjusted R^2, the more important the correlation between the LPP Swiss Index and the HFR strategy becomes. Moreover, we show that the parameters of the non-linear regressions are stable[11] throughout time[12].

HFR Weighted Composite Index (HFRWC) analysis

This index is an equally weighted index of all Hedge Funds based on the HFR database. It is long biased. Exhibit 2 below shows the local regression obtained with the Loess Fit technique. We observe that the payoff of the HFR Weighted Composite Index is concave compared to the LPP[13] index. The straight line in Exhibit 1 corresponds to a 100% investment in the LPP index, considering that our reference asset is the LPP index. Furthermore, Exhibit 2 suggests that the explanatory power of the linear regression can be improved upon by using a quadratic regression.

Exhibit 2[14]

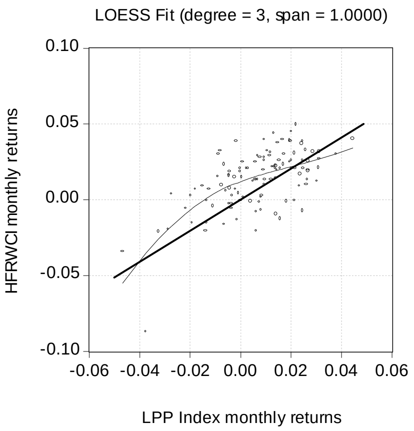


Exhibit 2 shows that the HFRWC generates a slightly improved payoff, as compared to the LPP index, between -4% and +2% monthly LPP index returns. The appendix shows the result of a quadratic regression between the HFR weighted composite index and the LPP index.

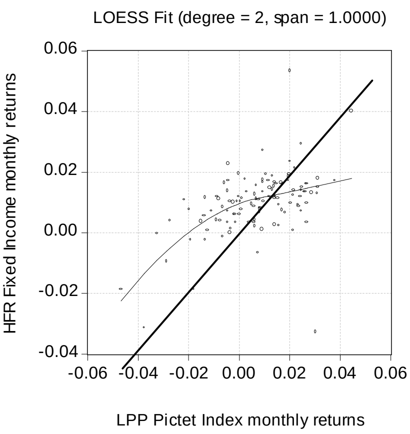
The explanatory power of the regression (ie. adjusted R^2 =0.42) is good. It is equivalent to a correlation coefficient of 0.65. According to the Chow test[15], the coefficients of the regression above are stable throughout the time period at 99%.

HFR Total Fixed Income Index (HFRFI) analysis

As mentioned by Fung and Hsieh (1999)[16], the fixed income arbitrage strategy produces stable returns with low dispersions[17]. They argue that Arbitrage Fixed Income managers are not capturing mispricings, but that they sell economic disaster insurance. When the market is quiet, the managers perform well and poorly in volatile markets. For example, when the liquidity dried up in the months September, October and November 1998, the HFR Total Fixed Income Index lost –3.1%, -1.8% and –3.2% respectively.

This fact is confirmed by Exhibit 3, where the slope for LPP returns below -1% of the regression, dramatically increases.

Exhibit 3



Based on Exhibit 2 and on the statistical Chow test, we performed two different regressions: one regression for index returns between –5%/month and +0.5%/month and another regression for index returns between 0.5%/month and 4.5%/month. The first regression has a quadratic form and the second a linear one with a slope of 0.2. A significant quadratic regression, with an adjusted R^2 of 47%, below a return of 0.5%, means that the investor becomes more and more exposed, when the index returns turn negative. This strategy can be seen as buying 0.2 LPP Indexes and selling put options with strikes further and further out-of-the-money[18].

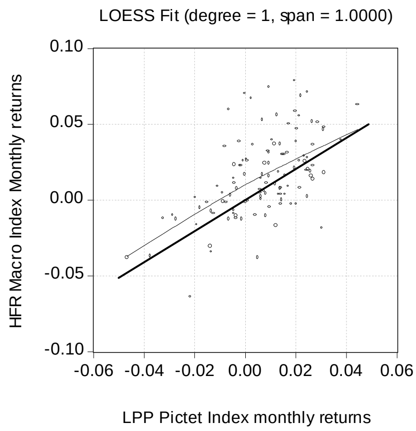
The appendix shows the explanatory power of the first quadratic regression to be good (47%). The correlation coefficient, considering only the negative returns, equals 0.69.

HFR Macro Index (HFRMA) analysis

The macro managers anticipate market movements by using top-down approaches. Historically, they achieve high yearly returns. Furthermore, they argue that their investments have low linear correlations with traditional instruments.

Exhibit 4 shows that, if there is a significant relationship, it should be a linear one. We performed a linear regression between the HFRMA Index and the LPP index and found a significant relationship with an adjusted R^2 of 0.29[19]. The constant of the linear regression is equal to 0.009 and the coefficient of the LPP index equals to 0.904. This means that, by investing in a macro Hedge Fund, the investor will be exposed to 0.904 of the returns of the LPP index[20].

Exhibit 3

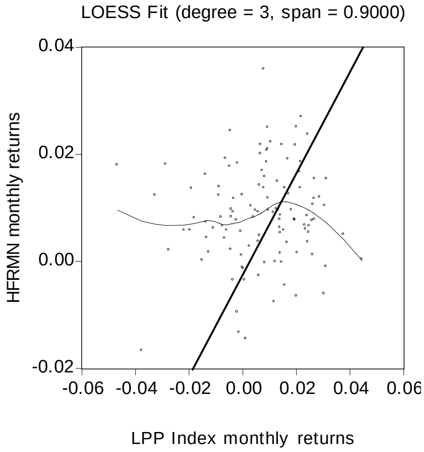


HFR Market Neutral (HFRMN) analysis

By definition, this strategy should have a beta of zero with the market. The market could be equity, bond, commodity, currency, real estate, private equity or markets. By trading on the long and short sides, in theory, they should neutralize their exposure to each of these different markets. We will show that this theoretical beta of zero, in a linear regression, is valid only on the LPP negative side.

Exhibit 4 shows that the relation between HFRMN and the LPP Index is not defined. When the LPP Index gets monthly returns higher than ~1.7%, the HFRMN Index gets returns more and more smaller (all the part on the left of the straight line). When the LPP Index returns are negative, the HFRMN Index performs very well and is always positive.

Exhibit 4



In order to see the relation between LLP pension fund index and Market Neutral strategy, a regression to the power three is done.The appendix shows that the relation between both indices is small (adjusted R^2 = 1.2%). All the coefficients of the regression to the power three are significant at 95%, as the absolute t-stat are higher than 1.96.

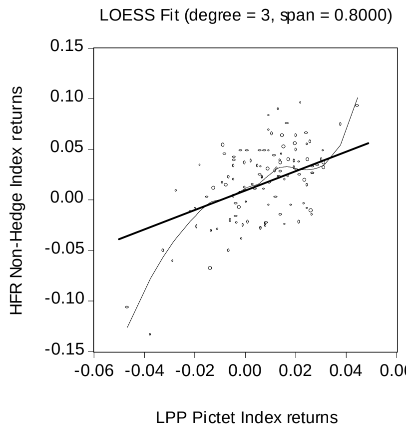
In conclusion, this strategy provides really good diversification for a swiss pension fund with a non-linear correlation of 0.11, no exposure at all on the downside, an annual volatility of 3.2%[21] and an historical annual return of 10%.

HFR Equity Non-Hedge (HFRNE) analysis

This strategy has similar features[22] as the HFR Equity Hedge Index, from a statistical point of view, except that the HFRNE return distribution is more dispersed.

Exhibit 5 shows the payoffs of the HFRNE index as compared to the LPP Index. By using a polynomial third degree regression, the shape of the relation is concave for negative LPP returns and convex for positive LPP returns. The slope of the concave regression varies between 2.6 and 1.0. This means that for negative LPP Index returns, each -1% in the former index leads to losses which are 2.6 times higher. On the other hand, for positive LPP returns, it is possible to increase returns by investing in the HFRNE-Index. So, the HFRNE-Index can be seen as a long position in the LPP-Index, some long out-of-the money calls and some short out-of-the money puts.

Exhibit 5



In the appendix, we show the relationship between both indices with a third degree regression. The correlation coefficient of this regression is equal to 0.63. All the parameters of this third degree regression are significant at 95%[23].

These concave and convex payoffs can be explained by the managers investment decisions. As the market drops, the manager incurs losses according to his long position and to the leverage (ie. short puts finance the long calls). As the market rises, the leverage of his position leads to very high returns.

The vision to invest in such a strategy is either very bullish or stable.

HFR Convertible Arbitrage Index (HFRCA) analysis

The managers following these strategies are arbitraging convertible instruments. Note that this strategy is highly exposed to credit- and leverage risk.

In Exhibit 6, we rank the index returns (here the LPP) from the lowest negative to the highest positive among the sample 1990-1999. Then, the corresponding HFRCA returns are added. One can see that the HFRCA Index returns are more or less stable, despite a few bad deals during market turmoils. There are only four negative months for the HFRCA, which corresponds exactly with the worst LPP returns. Except for that, as Exhibit 6 shows, the returns of the HFRCA are almost visually stable throughout time.

Exhibit 6

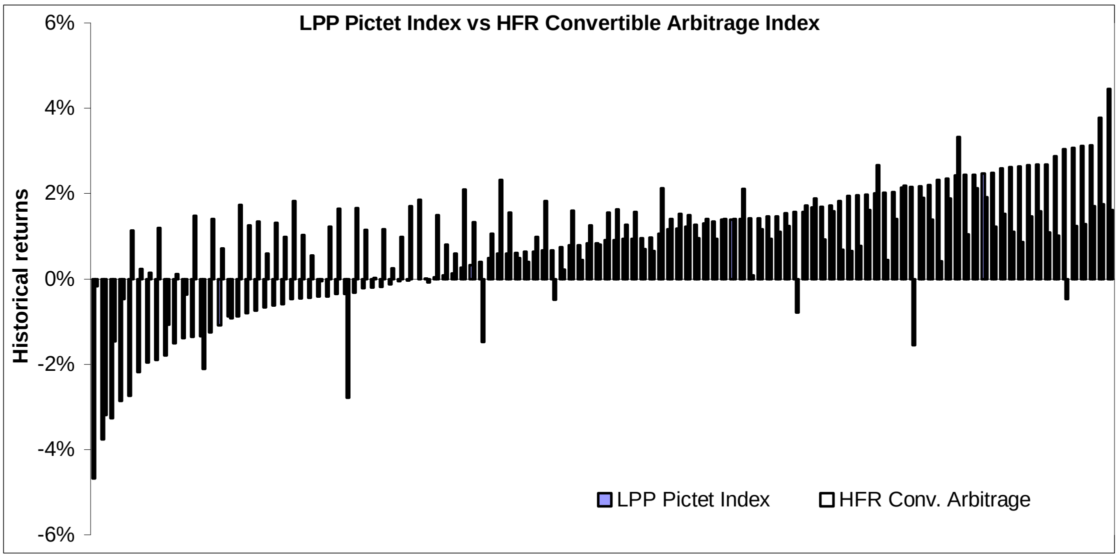
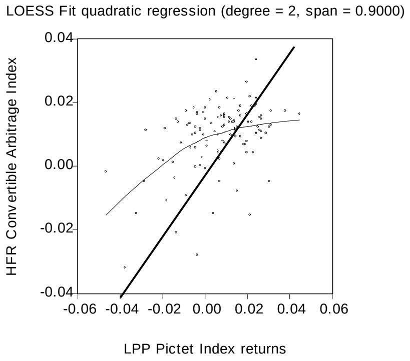


Exhibit 7 provides another view of the relationship between the two indices. The bent line is a quadratic regression with a smooth coefficient of 0.9. When the bent line is above the straight line, then in terms of returns, the investor will be better off buying the HFRCA Index than by buying the LPP Index.

Exhibit 7



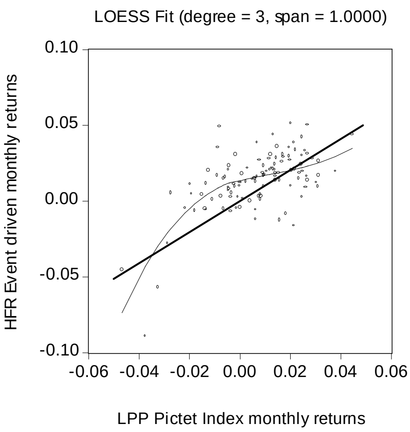
When the LPP Index returns rise (straight line), then the HFRCA returns do not move in the same manner (bent line). The exposure to negative LPP Index returns is low, since the slope of a local regression with only negative LPP returns is less than 1. The results in the appendix confirms the fact that a regression is not powerful. We obtain a quadratic regression with an adjusted R^2 of 0.20[24]. All the coefficients of the regression are strongly significant. This leads to the conclusion that the relation between the LPP Index and the HFRCA is concave (as shown in Exhibit 7), but the power of the relation is poor.

HFR Event-Driven (HFRED) analysis

The managers using this strategy are investing in significant transactional events such as spin-offs, bankruptcies, recapitalizations and share buy-backs. The instruments used are short and long stocks, debts and options. This will explain the strong significant non-linear regression obtained thereafter.

The payoffs in Exhibit 8 indicate that a quadratic regression is not appropriate. The relationship between both payoffs starts changing, when LPP monthly returns are above 2%.

Exhibit 8



As shown in the appendix the local regression between both indices is well explained with a polynomial third degree regression. The adjusted R^2 is equal to 0.46 and the correlation coefficient is equal to 0.69, which gives a high explanatory power to the regression. All the coefficients of the regression are significant at 95%, when the t-statistic is higher than ±1.96. The regression coefficients of the LPP^2 and LPP^3 are high. Their signs prove the increasing exposure to negative independent variable values.

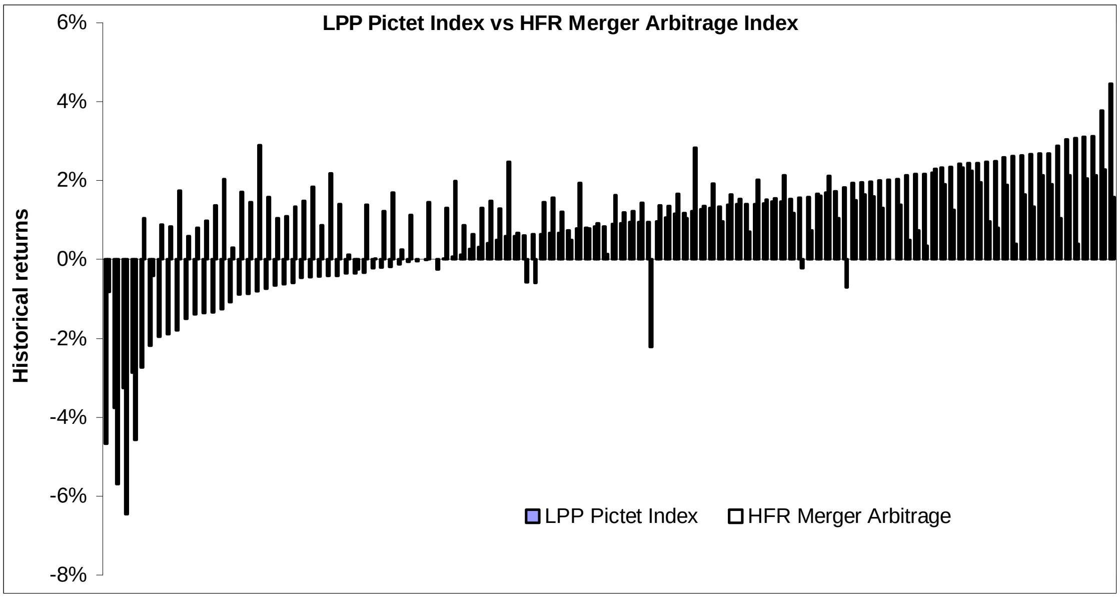
The parameters are not stable or consistent between the two chosen sub-samples[25]. Nevertheless, the parameters of the two sub-sample regressions are near (ie. in terms of the confidence interval) those obtained with a regression over all the samples.

HFR Merger Arbitrage (HFRMAR)

HFR Merger Arbitrage managers are investing in leveraged buy-outs, mergers and hostile take-overs.

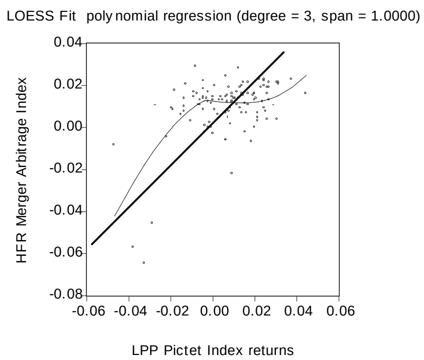
The ranked graph with respect to the LPP Index returns, in Exhibit 9, shows that the returns of the HFRMAR Index are stable, despite three extremely negative returns. This is due to the fact that this strategy is sensitive to important market shocks (liquidity risk).

Exhibit 9



The graph in Exhibit 10 shows a kind of concave relationship with a bump. The explanation of the concave relationship is, that the managers invested in some mispriced securities. If they prove to be wrong, the losses may be really high.

Exhibit 10



In order to fit above the relationship, a third degree regression is performed in the apapendix. The coefficients are significant at 95% and the adjusted R^2 is equal to 0.29. Like Exhibit 10, the regression in the appendix shows that the returns of the HFRMAR become more and more negative with respect to negative LPP Index returns.[26]

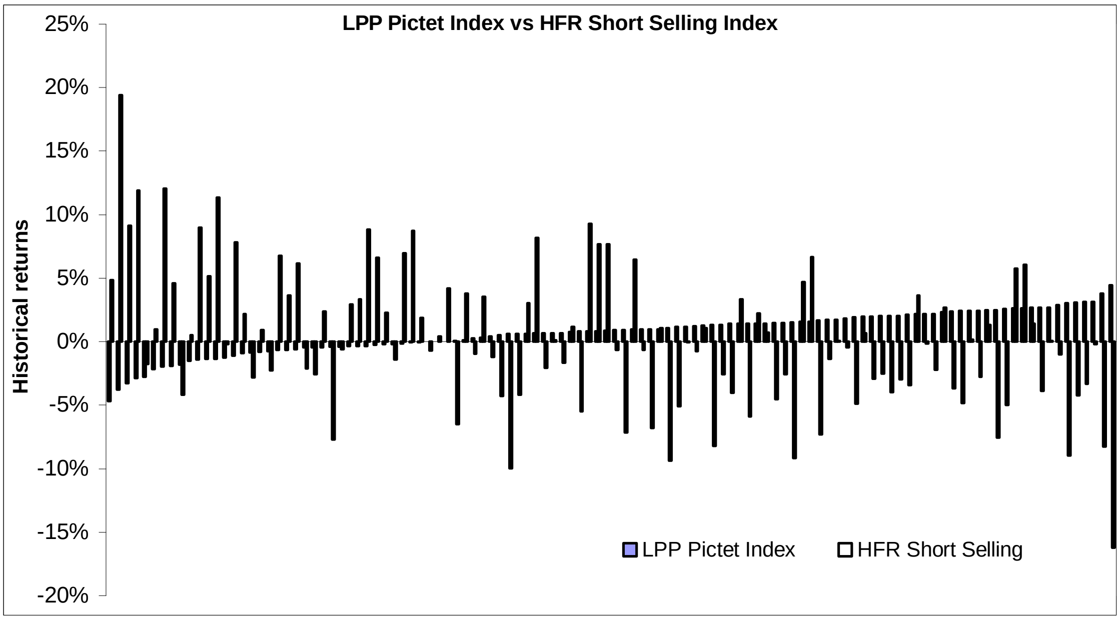
Thus, this strategy is not to diversify the risks of a Swiss pension fund during strong negative LPP returns.

HFR Short Selling (HFRSS) analysis

A priori, the HFR Short Selling strategy should pay-off when a global index like the SP&500 or the MSCI have negative returns. This should be reflected, as well, through the payoff of the LPP Index returns.

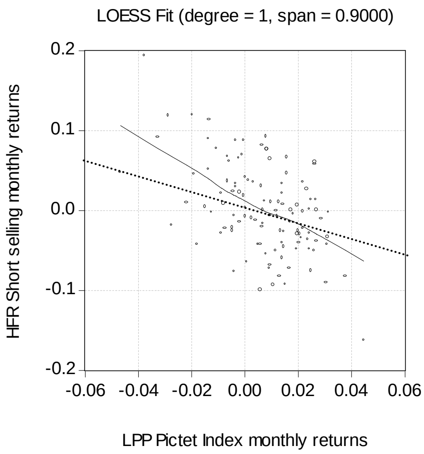
Exhibit 11 confirms that when the LPP Index records positive returns, the HFRSS Index tends to have strong negative ones and inversely.

Exhibit 11



The local regression is shown in the following graph (fig. 12). The relationship is linear and negative. The dashed line represents a 100% investment in the LPP Index and the straight line represents the investment in the HFRSS. In the CAPM, the beta of the HFRSS, with respect to the LPP Index, would be around –1.7.

Exhibit 12



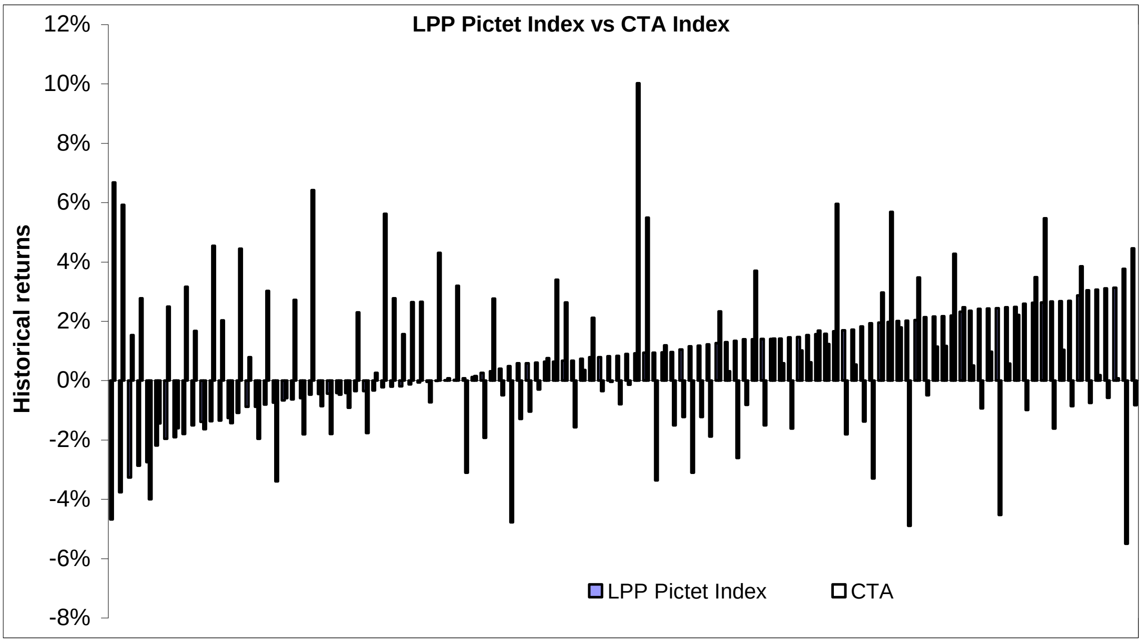
The linear regression gives a power of explanation of 26% in the appendix. The coefficients are significant at 99%. The linear relationship between both indices with a slope of –1.76 is negative. Thus, the HFRSS can be seen as selling 1.76 futures on the LPP Index[27].

It is only interesting to invest in the short selling strategy, if traditional markets earn bad returns. During positive LPP Index returns, this strategy shows a poor performance[28].

CTA analysis

Besides the Hedge Funds managers, the Commodity Trading Advisors are investing in futures too. To analyze this strategy, we use the Barclays CTA Index. This strategy is also called 'Trend Following'. Fung and Hsieh (1999)[29] analyzed this strategy and concluded that it is similar to a lookback call and a lookback put[30] on the SP&500. They stress, however, that there exists no relations, in terms of R^2, between the major indices and CTA returns. We find exactly the same results between the LPP Index and the CTA Index, as shown in Exhibit 13. Positive and negative CTA returns seem to occur randomly along the sorted LPP returns.

Exhibit 13

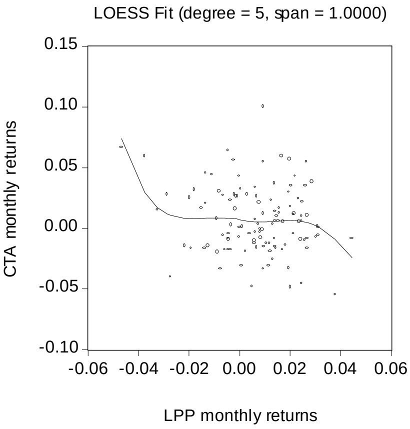


Source: authors, Barclays Index.

The monthly mean return, for the period January 1990-June 1999, is equal to 0.67% (8% per year)[31]. This is low when compared to the other Hedge Funds strategies. This low monthly return could be explained by the price being paid for these low moving correlations.

The local regression technique, in Exhibit 14 shows that the payoffs of the CTA can be seen as equal to long puts OTM[32] and short calls OTM. One sees as well, a really high dispersion of the returns on both sides of the broken line. This tells us that it will be difficult to find a relation between the CTA and the LPP Index.

Exhibit 14



In order to find significant regression coefficients between the CTA and the LPP Index, a third degree regression is carried out (see appendix). The power of the regression is poor, as the adjusted R^2 equals 5.3% and the correlation coefficient comes to 0.25[33]. This result is different from the one obtained by Fung and Hsieh (1999)[34] for two reasons. Firstly, we use the LPP Index instead of the S&P500 Index, which they did. Secondly, we are using end of month returns, instead of intra-month returns.

In conclusion, this strategy provides a good diversification, but low returns when compared to the Hedge Funds strategies.

Conclusion

We have just seen that four out of ten Hedge Funds strategies[35] have concave payoffs. This is like selling options. Therefore, these strategies are capped, when high LPP[36] returns occur. We have also seen that six out of ten have concave payoffs on the downside[37]. We have also observed that diversification benefits tend to disappear in case of extremely negative LPP Index returns, except in case of short-selling, market neutral, CTA and convertible arbitrages.

Generally speaking positive LPP returns do not explain a lot about the Hedge Funds strategies returns. This can be explained by the fact that the Hedge Funds managers reduce their risks, when reaching a positive monthly return[38]. So a manager has to reach a positive return level which he has set for himself. As long as he has not reached this target return, he takes risks and is exposed to the underlying market, which in our case is represented by the LPP Index.

Convertible Arbitrage strategy has a concave payoff with respect to the LPP index over the period 1990-1999, with an average historical annual return of 11%. The slope of the concavity is never higher than one which means that on the downside, the strategy will never loose more than the market. Market Neutral strategy has almost no relation with the LPP index over the same period. The strategy offers a good protection on the downside with a historical annual return of 10.4%. The CTA strategy is like a negative third degree regression with respect to the LPP Index (see Exhibit 14). This means that the Swiss pension fund will have a negative correlation during negative LPP returns and a negative correlation during positive LPP returns. The average historical annual return is 8%

Thus, by means of the payoff analysis, assuming that the investor is a Swiss pension fund, only three strategies will give a diversification effect during market downturns[39]: Convertible Arbitrage, Market Neutral and CTA. Other strategies are interesting in term of risk-returns as soon as the market is not volatile. This is due to the fact that the payoffs are similar to short option positions.

We would add Hedge Funds to a diversified Swiss pension fund portfolio under four conditions:
- invest in Convertible Arbitrage, Market Neutral and CTA[40]
- diversify among Hedge Funds in order to decrease the volatility, negative skewness and kurtosis and then test the Hedge Fund portfolio with the same techniques we developed in this article
- combine equities, bonds and Hedge Funds in a portfolio by minimizing volatility, skewness and kurtosis
- each Hedge Fund has followed a qualitative analysis

Appendix 1

Exhibit 1

Regression between HFR Weighted Index & LPP

HFR Weighted composite index = 0.011 + 0.784 * LPP - 10.638 * LPP^2

Sample: 1990:01 1999:06

Included observations: 114

Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	**0.011517**	0.001656	6.955799	0.0000
LPP	**0.784128**	0.084616	9.266952	0.0000
LPP^2	**-10.63889**	3.486322	-3.051609	0.0028
R-squared	0.433121	Mean dependent var		0.013725
Adjusted R-squared	**0.422907**	S.D. dependent var		0.019331
Sum squared resid	0.023937	Schwarz criterion		-5.506000
Log likelihood	320.9463	F-statistic		42.40442

Exhibit 2

Regression between HFR Fixed Income & LPP

HFR Fixed Income = 0.01 – 17.37 * LPP^2

From -5% to 0.5% LPP returns

Newey-West HAC Standard Errors & Covariance (lag truncation=3)

	Coefficient	Std. Error	t-Statistic	Prob.
LPP^2	**-17.37522**	1.794328	-9.683416	0.0000
R-squared	0.476186	Mean dependent var		0.005480
Adjusted R-squared	**0.476186**	S.D. dependent var		0.009996
Sum squared resid	0.002303	Schwarz criterion		-6.957787

Regression between HFR Fixed Income & LPP

HFR Fixed Income = 0.009 + 0.212 * LPP

From 0.5% to 4.5% LPP returns

Newey-West HAC Standard Errors & Covariance (lag truncation=3)

	Coefficient	Std. Error	t-Statistic	Prob.
LPP	**0.212654**	0.085554	2.485608	0.0154
R-squared	0.034381	Mean dependent var		0.012691
Adjusted R-squared	**0.034381**	S.D. dependent var		0.010161
Sum squared resid	0.006779	Schwarz criterion		-6.328731

Exhibit 3

Regression between HFR Macro and LPP

HFR MACRO = 0.009 + 0.904 * LPP

Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	**0.009544**	0.002090	4.566153	0.0000
LPP	**0.904237**	0.097596	9.265064	0.0000
R-squared	0.303892	Mean dependent var		0.015918
Adjusted R-squared	**0.297677**	S.D. dependent var		0.026682
Sum squared resid	0.056003	Schwarz criterion		-4.697588

Exhibit 4

Regression between HFR Market neutral & LPP

HFR Market neutral = 0.08 + 0.16 * LPP -141 * LPP^3

Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	**0.008068**	0.001036	7.784328	0.0000
LPP	**0.163841**	0.065820	2.489228	0.0143
LPP^3	**-141.3928**	59.27455	-2.385387	0.0188
R-squared	0.030264	Mean dependent var		0.008758
Adjusted R-squared	**0.012792**	S.D. dependent var		0.009033
S.E. of regression	0.008975	Akaike info criterion		-6.562767
Sum squared resid	0.008941	Schwarz criterion		-6.490762
Log likelihood	377.0777	F-statistic		1.732091
Durbin-Watson stat	1.714620	Prob(F-statistic)		0.181661

Exhibit 5

Regression between HFR Non-Hedge & LPP

HFR Non-Hedge = 0.01 + 0.99 * LPP –17.84 * LPP^2 + 561.20 * LPP^3

Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	**0.013383**	0.003370	3.971117	0.0001
LPP	**0.991887**	0.277369	3.576055	0.0005
LPP^2	**-17.84362**	5.264458	-3.389451	0.0010
LPP^3	**561.2039**	222.0178	2.527743	0.0129
R-squared	0.412830	Mean dependent var		0.016654
Adjusted R-squared	**0.396816**	S.D. dependent var		0.038345
Sum squared resid	0.097556	Schwarz criterion		-4.059469
Log likelihood	240.8621	F-statistic		25.77977

Exhibit 6

Regression between HFR distressed & LPP

HFR Distressed = 0.01 + 0.63 * LPP – 11.94 * LPP^2

Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	**0.012651**	0.001760	7.189650	0.0000
LPP	**0.639249**	0.083782	7.629914	0.0000
LPP^2	**-11.94310**	3.566228	-3.348945	0.0011
R-squared	0.306604	Mean dependent var		0.013432
Adjusted R-squared	**0.294111**	S.D. dependent var		0.019230
Sum squared resid	0.028975	Schwarz criterion		-5.315008
Log likelihood	310.0598	F-statistic		24.54089

Exhibit 7

Regression between HFR Convertible & LPP

HFR Convertible Arbitrage = 0.008 + 0.300 * LPP –4.570 * LPP^2

Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	**0.008528**	0.000850	10.03619	0.0000
LPP	**0.300530**	0.044757	6.714636	0.0000
LPP^2	**-4.570266**	1.660964	-2.751575	0.0069
R-squared	0.222253	Mean dependent var		0.009221
Adjusted R-squared	**0.208240**	S.D. dependent var		0.010415
Sum squared resid	0.009533	Schwarz criterion		-6.426711
Log likelihood	373.4268	F-statistic		15.85997

Exhibit 8

Regression between HFR Event driven & LPP

HFR Event driven = 0.01 + 0.53 * LPP –14.36 * LPP^2 + 298.93 * LPP^3

Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	**0.013278**	0.001820	7.294398	0.0000
LPP	**0.530781**	0.165153	3.213869	0.0017
LPP^2	**-14.36354**	3.396376	-4.229078	0.0000
LPP^3	**298.9357**	131.3550	2.275785	0.0248
R-squared	0.479332	Mean dependent var		0.013521
Adjusted R-squared	**0.465132**	S.D. dependent var		0.019568
Sum squared resid	0.022529	Schwarz criterion		-5.525071
Log likelihood	324.4014	F-statistic		33.75569

Exhibit 9

Regression between HFR Merger Arbitrage & LPP

HFR Merger Arbitrage = 0.01 –9.84 * LPP^2 +402.55 * LPP^3

Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	**0.011753**	0.001149	10.23296	0.0000
LPP^2	**-9.847384**	4.377615	-2.249486	0.0265
LPP^3	**402.5515**	114.0146	3.530700	0.0006
R-squared	0.312327	Mean dependent var		0.010004
Adjusted R-squared	**0.299936**	S.D. dependent var		0.013665
Sum squared resid	0.014511	Schwarz criterion		-6.006502
Log likelihood	349.4749	F-statistic		25.20696

Exhibit 10

Regression between HFR Short selling & LPP

HFR Short selling = 0.014 – 1.76 * LPP

Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	**0.014653**	0.004484	3.267830	0.0014
LPP	**-1.763566**	0.266439	-6.619030	0.0000
R-squared	0.265560	Mean dependent var		0.002220
Adjusted R-squared	**0.259002**	S.D. dependent var		0.055669
Sum squared resid	0.257196	Schwarz criterion		-3.173147
Log likelihood	185.6056	F-statistic		40.49710

Exhibit 11

Regression between CTA & LPP

CTA = 0.007 – 398 * LPP^3

Newey-West HAC Standard Errors & Covariance (lag truncation=4)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	**0.007966**	0.002388	3.335650	0.0012
LPP^3	**-398.1878**	115.8161	-3.438103	0.0008
R-squared	0.062217	Mean dependent var		0.006657
Adjusted R-squared	**0.053844**	S.D. dependent var		0.027613
Sum squared resid	0.080800	Schwarz criterion		-4.331004
Log likelihood	251.6034	F-statistic		7.430565

Appendix 2

We will show that the classical constant correlation coefficient underestimates the relation between two assets as soon as the relation between them is no linear. This is often the case in Hedge Funds. We will see 2 cases:

- a hedge fund which is exposed to volatility (ie. long only strategy) with payoffs
$Y = X^3 + \varepsilon$
- a hedge fund which has convex payoffs (ie. long puts and long calls strategy) following
$Y = X^2$
- a hedge fund which has a concave or convex payoffs (ie. convertible arbitrage strategy) following
$Y = aX^2 + \varepsilon$

where Y is a portfolio with options and X is a classical index. The returns of the index X follow a normal distribution N(0,1).

The constant correlation between the two assets X and Y is given by

$$\rho = \frac{E(xy) - E(x)E(y)}{\sqrt{\left(E(x^2) - E(x)^2\right)\left(E(y^2) - E(y)^2\right)}}$$

The non constant correlation between the two assets X and Y is given by

$$\rho_{non\,\text{constant}} = \rho(X,Y) = \max_f \left(f(X), Y\right)$$ [41]

First case

The portfolio Y can be replicated with a long index X, short puts and long calls on the index X (see Equity non-hedge strategy for a real example):

$$y = x^3 + \varepsilon \qquad (1)$$

First let's assume the relation is well defined, ε = 0. Thus

$$y = x^3 \qquad (2)$$

Then, the constant correlation is equal to

$$\rho\,(y,x) = \frac{E(x^4) - E(x)E(x^3)}{\sqrt{\left(E(x^2) - E(x)^2\right)\left(E(x^6) - E(x^3)^2\right)}} \underbrace{=}_{\text{x is centered}} \frac{E(x^4) - 0}{\sqrt{E(x^2)E(x^6) - E(x^2)E(x^3)^2 - E(x)^2E(x^6) + E(x)^2E(x^3)^2}}$$

$$\rho = \frac{E(x^4)}{\sqrt{E(x^2)}\sqrt{E(x^6) - E(x^3)^2}} = \frac{E(x^4)}{\sigma_x \sigma_{x^3}} \underbrace{=}_{x\sim N(0,1)} \frac{3}{1 * \sqrt{15}} = \frac{3}{\sqrt{15}} = 0.77$$

with

$$\sigma_{x^3} = \sqrt{\sigma^2_{x^3}} = \sqrt{E(x^6)} = \sqrt{1*3*5} = \sqrt{15}$$

As we have assumed that the process X follows a normal distribution (0,1) with no random term ε, the constant correlation between a X and Y is 0.77.

From (2), we have assumed that the process Y is driven by $Y = X^3$. Thus, the non constant correlation between Y and X^3 is

$$\rho_{non\,\text{constant}} = corr(y, x^3) = corr(x^3, x^3) = 1$$

There is a full deterministic relationship between X and Y. The constant correlation measures only the linear relation between the two process. This is why the constant correlation gives 0.77 and the non-constant correlation is 1.

Second case
Let's take an example where the constant correlation is equal to zero and there is a deterministic relationship between X and Y:

$$y = x^2$$

The constant correlation is equal to

$$\rho(x, x^2) = \frac{E(xx^2) - E(x)E(x^2)}{\sqrt{\left(E(x^2) - E(x)^2\right)\left(E(x^4) - E(x^2)^2\right)}} \underbrace{=}_{x\sim N(0,1)} \frac{0 - 0 * 1}{\sqrt{(1-0)(3-1)}} = 0$$

The constant correlation shows no relation between both distribution even though the Y asset is depending on the X asset. This is due to the fact that the constant correlation "tries" to find linear relation between X and Y. In this case, there is absolutely no linear relation between X and Y, but only a positive quadratic relation.

Third case
Now, we will show that the relation between two stochastic processes with random terms cannot be measured with the constant correlation coefficient. After a non-linear regression, one sees that the relation between two processes X and Y is of the form

$$y = ax^2 + \varepsilon$$

Assume a=1. The non-constant correlation between asset x^2 and asset Y is equal to

$$\rho(x^2, x^2 + \varepsilon)_{non\ \mathrm{constant}} = \frac{E(x^2(x^2 + \varepsilon)) - E(x^2)E(x^2 + \varepsilon)}{\sqrt{E(x^4) - E(x^2)^2}\sqrt{E(x^4 + 2x^2\varepsilon + \varepsilon^2) - E(x^2 + \varepsilon)^2}}$$

Assuming the asset X is normally distributed N(0,1) for simplicity, the real correlation equals

$$\rho_{non\ \mathrm{constant}} = \frac{E(x^4 + x^2\varepsilon) - 1}{\sqrt{3-1}\sqrt{E(x^4 + 2x^2\varepsilon + \varepsilon^2) - 1}} = \frac{E(x^4) + E(x^2\varepsilon) - 1}{\sqrt{2}\sqrt{E(x^4) + 2E(x^2\varepsilon) + E(\varepsilon^2) - 1}}$$

$$= \frac{3 + E(x^2\varepsilon) - 1}{\sqrt{2}\sqrt{3 + 2E(x^2\varepsilon) + \sigma_\varepsilon^2 - 1}} = \frac{2 + E(x^2\varepsilon)}{\sqrt{2}\sqrt{2 + 2E(x^2\varepsilon) + \sigma_\varepsilon^2}} \underbrace{=}_{\substack{X^2 and\, \varepsilon \\ non\, correlated}} \frac{2 + E(x^2)E(\varepsilon)}{\sqrt{2}\sqrt{2 + 2E(x^2)E(\varepsilon) + \sigma_\varepsilon^2}}$$

$$\underbrace{=}_{E(\varepsilon)=0} \frac{2}{\sqrt{2}\sqrt{2 + \sigma_\varepsilon^2}} = \frac{\sqrt{2}}{\sqrt{2 + \sigma_\varepsilon^2}} \to 1 \quad \text{as}\ \sigma_\varepsilon \to 0$$

If ones computes the constant correlation coefficient between asset X [42]and asset Y, one finds (always assuming that a=1 and X ~ N(0,1))

$$\rho = \frac{E(x(x^2+\varepsilon)) - E(x)E(x^2+\varepsilon)}{\sqrt{E(x^2)-E(x)^2}\sqrt{E(x^4+2x^2\varepsilon+\varepsilon^2)-E(x^2+\varepsilon)^2}}$$

$$\rho = \frac{E(x^3+x\varepsilon)-0}{\sqrt{1-0}\sqrt{E(x^4+2x^2\varepsilon+\varepsilon^2)-E(x^2+\varepsilon)^2}} = \frac{E(x^3)+E(x\varepsilon)}{\sqrt{E(x^4)+2E(x^2\varepsilon)+E(\varepsilon^2)-1}}$$
$$\underbrace{=}_{\substack{x^2 \text{ and } \varepsilon \\ \text{non correlated}}} \frac{0+E(x\varepsilon)}{\sqrt{3+2E(x^2)E(\varepsilon)+\sigma_\varepsilon^2-1}} \underbrace{=}_{E(\varepsilon)=0} \frac{E(x\varepsilon)}{\sqrt{2+\sigma_\varepsilon^2}} \underbrace{=}_{Footnote(1)} \frac{1}{\sqrt{2+\sigma_\varepsilon^2}}\rho_{x,\varepsilon}\sigma_\varepsilon$$

Assuming the asset X ~ N(0,1) and $\rho_{X,\varepsilon} \leq 1$. Thus,

$$\rho \leq \frac{\sigma_\varepsilon}{\sqrt{2+\sigma_\varepsilon^2}} \to 0 \quad \text{as } \sigma_\varepsilon \to 0$$

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[1] See appendix 2
[2] The skewness measures the asymmetry of a distribution. A normal distribution has a skewness of zero.
[3] The kurtosis measures returns which are highly positive or highly negative with respect to the other returns. In other words, the kurtosis measures if the distribution has fat-tailed. A normal distribution has a kurtosis of 3 and an excess kurtosis of zero.
[4] The LPP Index or BVG Index is the Index constructed by Pictet & Cie (Geneva) and represents the Benchmark Index for a Swiss institutional investor. Typically, this index does not include more than 30% of the SPI, 25% of the MSCI, 20% of the Salomon Brother Global Bond Index.
[5] A negative skewness implies that the distribution has a long left tail. Risk averse investor does not like negative skewness.
[6] Only the Macro, the Short Selling and the CTA strategies have a normal distribution based on the Jarque-Berra statistics.
[7] HFR Weighted Composite Index, HFR Fixed Income, HFR Convertible Arbitrage, HFR Event Driven
[8] We have excluded Emerging markets which are not well defined in term of strategy, excluded Equity hedge strategies which are similar to Equity non-hedge, but with lower volatility and lower kurtosis.

[9] For more information on local regression analysis, see Chambers, Hastie, Statistical models in S, 1992, Chapter 8, Wadsworth & Brooks.
[10] We use Newey-West regression, which adjusts for autocorrelation and heteroskedasticity.
[11]
[12] To do that, the period January 1990-June 1999 is divided in two equal sub-periods. A Chow-test which follows an F-distribution with 3 and 122 degrees of freedom is performed.
[13] Remember that the LPP index is the index constructed by Pictet & Cie (Geneva) and represents the benchmark index for a Swiss institutional investor. Typically, this index consists of not more than 30% SPI, 25% MSCI, 20% Salomon Brother Global Bond Index, 100% Swiss bonds.
[14] One interpretation of this graph is that the investor would have been better of investing in the Hedge Fund Strategy, when the concave curve was situated above the flat line.
[15] The Chow-test, which measures the stability of the regression between two sub-periods (ie. Jan.1990-Sept.1994 and Oct.1994-June 1999), is equal to 3.06. The critical level of this Chow-test F(3,122) is 3.95 at 99%.
[16] W.Fung, D.H.Hsieh, A primer on Hedge Funds, March 1999, Working paper, Duke University.
[17] The empirical Value-at-Risk at the 95% level for the HFR Fixed Income Arbitrage and for the HFR Total Fixed Income is –2.58% and -0.66% respectively.
[18]On average as the adjusted R^2 is not equal to 100%.
[19] Fung and Hsieh, 1999, A primer on Hedge Funds, found that the payoffs of the macro strategies can be seen as a long position in the SP500, short calls in-the-money and long put positions. But they do not provide a local linear coefficient in order to prove the statistical validity of their conclusions.
[20] On average as the adjusted R^2 is not equal to 100%.
[21] The conversion between monthly and annual volatility is valid as the distribution of HFRMN is normal (Jarque-Berra = 1.22).
[22] i.e. in terms of mean and linear correlation.
[23] The stability test of the parameters of the polynomial third degree expansion regression between two sub-samples of equal size gives a Chow-test F(3,122) of 0.69. The critical F is equal to 3.95 at 99%. As 0.69<3.95, we cannot reject the hypothesis that the parameters are equal through time. The above regression's parameters are stable or consistent through time according to the Chow test.
[24] The above regression's parameters are stable or consistent through time according to the Chow test . The stability test of the parameters of the quadratic regression between two sub-samples of equal size gives a Chow-test F(3,122) of 0.72. The critical F is equal to 3.95 at 99%. As 0.72<3.95, we cannot reject the hypothesis that the parameters are equal.
[25] The stability test of the parameters of the quadratic regression between two sub-samples of equal size gives a Chow-test F(3,122) of 5.09. The critical F is equal to 3.95 at 99%. As 5.09>3.95, we reject the hypothesis that the parameters are equal through time.
[26] The coefficient sign of the independent squared and power three variables are respectively negative and positive.
[27] This is true under two conditions. Firstly the LPP can be replicated and secondly the LPP explains 100% of the HFRSS.
[28] Remember, in table 3, the monthly average return of the HFRSS is equal to 0.22%. During the year 2000, as the world market was negative in the course of the second part of the year, the HFRSS gained 4.8%.
[29] W.Fung, D.A.Hsieh, 1999, A risk neutral approach to valuing trend following strategies, Duke University, Working paper.
[30] A lookback call is a normal call option but the strike depends on the minimum stock price reached during the life of the option. A lookback put is a normal put option but the strike depends on the maximum stock price reached during the life of the option.
[31] In 2000, the CTA returns was –1.8% and 0.2% in 2001.
[32] Out-of-the-money
[33] Square root of 0.062.
[34] They found a straddle payoff.
[35] HFR Weighted Composite Index, HFR Fixed Income, HFR Convertible Arbitrage, HFR Event Driven
[36] The LPP Index is the index constructed by Pictet & Cie (Geneva), which represents the benchmark index for a Swiss institutional investor. Typically, this index consists of not more than 30% SPI, 25% MSCI, 20% Salomon Brother Global Bond Index.
[37] Same as four above, but add HFR Equity Non-hedge, HFR Merger Arbitrage
[38] See Brown, Goetzmann, Park, 1997
[39] for annual returns higher than 8%
[40] CTA have shown a performance of 4.2% in 2000 and 1.9% in 2001, Convertible 25.6% in 2000 and 13.4% in 2001, Market Neutral 15% in 2000 and 7.8% in 2001 (source CSFB Tremont index).
[41] Heinz Mueller, UBS

[42] $\rho_{x,\varepsilon} = \dfrac{E(x\varepsilon) - E(x)E(\varepsilon)}{\sigma_x \sigma_\varepsilon} = \dfrac{E(x\varepsilon) - 0}{1 * \sigma_\varepsilon} = \dfrac{E(x\varepsilon)}{\sigma_\varepsilon} \Rightarrow E(x\varepsilon) = \rho_{x,\varepsilon}\sigma_\varepsilon$